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PAPER NO. 1278

Intertemporal Asset Pricing and the Marginal Rate of Substitution: Empirical Estimation and Testing

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September 1986

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ABSTRACT

The purpose of this paper is to develop empirical estimates of the marginal rate of substitution (MRS) and tests of the intertemporal capital asset pricing model (CAPM), without imposing additional restrictions on the data. The MRS is treated as an unobservable variable and a method of moments estimator is developed by observing that there is a very large cross-section of security returns available from which one can construct sample moments. We use the MRS estimates to test the restrictions implied by the intertemporal CAPM and the results generally support the model. We also find that the estimates of the MRS differ substantially from those implied by the empirical versions of the consumption-based CAPM.

INTERTEMPORAL ASSET PRICING AND THE MARGINAL RATE OF SUBSTITUTION: EMPIRICAL ESTIMATION AND TESTING

In this paper, we develop empirical estimates of the marginal rate of substitution (MRS) and use the estimates to test the intertemporal capital asset pricing model (CAPM). The advantage of this approach is that we do not require additional assumptions to test the intertemporal CAPM. Tests of intertemporal CAPM's have followed two approaches. The most common one has been to use a consumption-based CAPM in which consumption data and a particular utility function are used to measure the marginal utility of real consumption. Examples of this approach can be found in Hansen and Singleton (1982, 1983), Dunn and Singleton (1983, 1984), Grossman and Shiller (1981), Ferson (1983), and Mankiw and Shapiro (1984). The empirical results have been generally negative: the models are rejected by the data on asset returns and the parameter estimates frequently result in implausible values. Mankiw and Shapiro find that consumption betas perform very poorly in the presence of betas estimated from the standard market model. A second approach has been to treat the MRS as an unobservable and impose additional assumptions on the joint distribution of asset returns and the MRS. Hansen and Singleton (1983) show that the joint lognormal distribution implies a restriction on the difference between the returns on two assets: specifically, expected excess returns are constant and excess returns should be unpredictable. Their tests with short-term interest rates and returns on large portfolios indicate rejection of these restrictions.

Several explanations for the poor performance of these empirical models have been mentioned in the literature. One argument is that we

need to measure the instantaneous consumption rate and that temporal aggregation of the published consumption data poses a serious problem. Another argument is that the time-additive separable utility function is too restrictive and a more complicated utility function is needed for consumption-based models. Garber and King (1983), for example, have shown that estimates of utility function parameters are biased if there is a random shock in the representative agent's utility of consumption function. The empirical results from a variety of studies suggest that the investment opportunity set (conditional distributions of asset returns) changes over time; specifically, the conditional means and variances of asset returns, interest rates, and the MRS vary over time. In the first section, we develop the empirical model and the method of moments estimator for the MRS. In the second section we present the results of the model. We use a large cross section of securities to estimate the MRS series and perform tests of the intertemporal CAPM on a subset of securities including a large stock portfolio. In addition, we provide some comparisons of our estimates of the MRS with the corresponding estimates from the log utility model and the consumption-based CAPM.

I. The Empirical Model and the Estimator of the MRS

Our approach to estimating and testing the intertemporal model is to treat the MRS as an unobservable variable and use both time series and cross-sectional data on returns to estimate the unobservable series. Let $J_w(t)$ be the marginal utility of real wealth, p_{it} be the real asset price, and d_{it} be the real dividend or cashflow at the end of period t. As

Breeden (1979) and others have shown, intertemporal asset pricing models imply the following asset pricing relation:

$$E_t[J_w(t)p_{it} - J_w(t+1)(p_{i,t+1}+d_{i,t+1})] = 0,$$

where E_t is the conditional expectations operator, conditional on information available at time t. Let λ_t equal the product of $J_w(t)$ and the consumption price deflator at time t, and we have the following relationship:

$$E_t[\lambda_t P_{it} - \lambda_{t+1}(P_{i,t+1}+D_{i,t+1})] = 0,$$

where P_{it} and D_{it} are price and dividends in nominal terms (nominal \$), for security i. In Appendix A, we show that these asset pricing relations can be derived from a rather weak set of assumptions. We have the following relationship for nominal returns:

$$E_t[\frac{\lambda_{t+1}}{\lambda_t}(1+R_{i,t+1}) - 1] = 0. \qquad (1)$$

The asset pricing relationship also applies to short-term securities that are riskless in nominal terms. For one-period risk-free interest rates, we have

$$\frac{1}{1+R_{F,t+1}} = E_t[\frac{\lambda_{t+1}}{\lambda_t}], \qquad (2)$$

where $R_{F,t+1}$ is the return known at time t for a one-period discount bond that matures at (t+1). This model is known in the literature as a MRS model. $\frac{\lambda_{t+k}}{\lambda_t}$ measures the marginal rate of substitution for a \$ between t+k and t. For convenience, we refer to λ_t as the marginal utility of wealth variable.

The asset pricing relation in (1) is a restriction on conditional moments, but the relationship implies the following restriction on unconditional moments;

$$E\left\{\left[\frac{\lambda_{t+1}}{\lambda_t}(1+R_{i,t+1}) - 1\right]\underline{z}_{it}\right\} = 0, \tag{3}$$

where $\underline{z}_{it}$ is a vector of information variables or instruments associated with security i known at time t. Our approach to estimating a time series of λ_t is based on the observation that there are many securities and information variables that must jointly satisfy the relationships in (3). We essentially use time series and the large cross-section of returns and information variables over time to identify and estimate the underlying marginal utility of wealth series.

Using equation (2) and the observation that the marginal utility of wealth variable should be positive, we develop the following model for λ_t:

$$\lambda_t = \left(\frac{\lambda_{t-1}}{1+R_{Ft}}\right)\eta_t,$$

where $\eta_t > 0$ and $E_{t-1}(\eta_t) = E(\eta_t) = 1$. The η_t series is serially uncorrelated, but not necessarily serially independent. Substituting this into (3), we get

$$E\left\{\left[\eta_t\frac{(1+R_{it})}{(1+R_{Ft})} - 1\right]\underline{z}_{i,t-1}\right\} = 0.$$

The following sample moments have expected values equal to zero:

$$\underline{u}_i \equiv \frac{1}{T}\sum_{t=1}^{T}\left[\eta_t \frac{(1+R_{it})}{(1+R_{Ft})} - 1\right]\underline{z}_{i,t-1}, \quad i = 1\ldots K$$

$$e_t \equiv \frac{1}{K}\sum_{i=1}^{K}\left[\eta_t \frac{(1+R_{it})}{(1+R_{Ft})} - 1\right], \quad t = 1,\ldots,T \tag{4}$$

where T is the number of time periods in the sample and K is the number of securities. The first set of sample moments consists of time-series moments, and the second set consists of cross-sectional moments. Our approach is to apply the intuition of Hansen's (1982) method of moments estimator to estimate the time series η_t, t=1,...T. We estimate the η_t series by simultaneously setting these sample moments as close as possible to zero. By letting K and T increase to infinity, we have a consistent estimator of $\underline{\eta}$, where $\underline{\eta}' = (\eta_1,\ldots,\eta_T)$.

The estimator that we employ is one that assigns an equal weight to each sample moment. An optimal estimator, following Hansen, would use the inverse of the covariance matrix for the sample moments, but the optimal estimator is not feasible in this application. $\underline{\eta}$ is estimated by minimizing the sum of the squares of the sample moments:

$$\min_{\underline{\eta}} F = \min_{\underline{\eta}} \sum_{t=1}^{T} e_t^2 + \sum_{i=1}^{K} \underline{u}_i'\underline{u}_i. \tag{5}$$

This estimator is linear and can be computed by solving the first-order conditions for the minimization problem.

$$\frac{\partial F}{\partial \eta_t} = (\eta_t \frac{1}{K} \sum_{i=1}^{K} \frac{1+R_{it}}{1+R_{Ft}} - 1)\ (\frac{1}{K} \sum_{j=1}^{K} \frac{1+R_{jt}}{1+R_{Ft}})$$

$$+ \frac{1}{T^2} \sum_{i=1}^{K} \{ \sum_{s=1}^{T} [\eta_s \frac{1+R_{is}}{1+R_{Fs}} - 1]\ \underline{z}'_{i,s-1}\ \underline{z}_{i,t-1}\ (\frac{1+R_{it}}{1+R_{Ft}}) \} = 0$$

$$t = 1, \ldots., T$$

This system of equations can be rewritten in matrix form as $A\underline{\eta} - \underline{b} = \underline{0}$, where $\underline{\eta}' = (\eta_1, \ldots., \eta_T)$. The t'th element of $\underline{b}$ is

$$\frac{1}{K} \sum_{i=1}^{K} \frac{1+R_{it}}{1+R_{Ft}} + \frac{1}{T^2} \sum_{i=1}^{K} (\frac{1+R_{it}}{1+R_{Ft}}) \sum_{s=1}^{T} \underline{z}'_{i,s-1}\ \underline{z}_{i,t-1}$$

The (j,m) off-diagonal elements of A are

$$\frac{1}{T^2} \sum_{i=1}^{K} (\frac{1+R_{ij}}{1+R_{Fj}})\ (\frac{1+R_{im}}{1+R_{Fm}})\ \underline{z}'_{i,j-1}\ \underline{z}_{i,m-1},$$

where j, m = 1,...,T and $j \neq m$. The j diagonal elements of A are

$$\frac{1}{T^2} \sum_{i=1}^{K} (\frac{1+R_{ij}}{1+R_{Fj}})^2\ \underline{z}'_{i,j-1}\ \underline{z}_{i,j-1} + (\frac{1}{K} \sum_{i=1}^{K} (\frac{1+R_{ij}}{1+R_{Fj}}))^2,$$

for j = 1,...,T. The resulting estimator is $\hat{\underline{\eta}} = A^{-1}\underline{b}$, where we require the inversion of a TxT symmetric matrix. The estimator is consistent as we let K, the number of securities, and T increase to infinity. As K and T increase, the sample moments are approaching their expected values and $\hat{\underline{\eta}}$ approaches the set of true parameter values. The consistency of $\hat{\underline{\eta}}$ is shown in Appendix B. Another view of the estimator is that we take a large fixed T and let K go to infinity.

With the estimator, we exploit the large cross-sections of security returns that are available.[1]

One can easily impose linear restrictions on this estimator. In the next section we present results for the estimator with two restrictions implied by the asset pricing model:

$$\frac{1}{T}\sum_{t=1}^{T} n_t = 1 \quad \text{and} \quad \frac{1}{T}\sum_{t=1}^{T} n_t \left(\frac{1+R_{mt}}{1+R_{Ft}}\right) = 1,$$

where R_{mt} is the return on a market portfolio of common stocks. Effectively, we require that two specific sample moments equal their respective expected values. This restricted estimator can be formed by solving a constrained minimization problem with LaGrange multipliers. The resulting estimator is

$$\hat{\underline{n}} = A^{-1}\underline{b} + \mu_1 A^{-1}\underline{i} + \mu_2 A^{-1}\underline{r}_m,$$

where $\underline{i}$ is a Tx1 vector of ones and $\underline{r}_m' = \left(\frac{1+R_{m1}}{1+R_{F1}},\ldots,\frac{1+R_{mT}}{1+R_{FT}}\right)$. μ_1 and μ_2 are LaGrange multipliers:

$$\mu_1 = \{(\underline{r}_m' A^{-1}\underline{r}_m)(T-\underline{i}'A^{-1}\underline{b}) - (i'A^{-1}\underline{r}_m)(T-r_m'A^{-1}\underline{b})\}/D$$

$$\mu_2 = \{(\underline{i}'A^{-1}\underline{i})(T-\underline{r}_m' A^{-1}\underline{b}) - (\underline{i}'A^{-1}\underline{r}_m)(T-\underline{i}'A^{-1}\underline{b})\}/D$$

where $\quad D = (\underline{i}'A^{-1}\underline{i})(\underline{r}_m' A^{-1}\underline{r}_m) - (\underline{i}'A^{-1}\underline{r}_m)^2$

For the optimal estimator, we need the covariance matrix for all the moments included (4). The elements of the covariance matrix for the time-series moments can be estimated by using sample variances and covariances from the time series data, but in most applications

the number of time-series moments (K-times the number of instruments for each security) will exceed T and the estimated matrix will be singular. The variances for the cross-sectional moments, the e_t's, include variances and covariances across securities and their estimation will be difficult. An alternative sub-optimal estimator would be to ignore the covariances and use simple variance estimates to weight each sample moment.[2]

Given estimates of $\underline{n}$, we can construct estimates of the MRS as follows:

$$\left(\frac{\hat{\lambda}_t}{\lambda_{t-1}}\right) = \frac{\hat{n}_t}{1+R_{Ft}} .$$

By normalizing and setting λ_0 equal to some arbitrary value, we can compute estimates of $\lambda_1,\lambda_2,\ldots,\lambda_T$. These latter estimates are unique up to a scalar transformation. The estimates $\left(\frac{\hat{\lambda}_t}{\lambda_{t-1}}\right)$ are unique. One strategy for testing the asset pricing relation in (3) is to use a large cross section of securities to estimate $\underline{n}$, and then use the estimated values for $\frac{\lambda_t}{\lambda_{t-1}}$ to test the relationship for a subset of securities. For each security the moment vector $\underline{u}_i$ has a coveriance matrix, $\frac{V_i}{T}$, where V_i is

$$V_i = E\left\{ \left[\left(\frac{\lambda_t}{\lambda_{t-1}}\right) (1+R_{it}) - 1\right]^2 \underline{z}_{i,t-1}\underline{z}_{i,t-1}' \right\}$$

The matrix V_i can be estimated from the corresponding sample moments:

$$\hat{V}_i = \frac{1}{T}\sum_{t=1}^{T} [(\frac{\hat{\lambda}_t}{\lambda_{t-1}}) (1+R_{it}) - 1]^2 \underline{z}_{i,t-1}\underline{z}_{i,t-1}' .$$

This estimate allows for the possibility of conditional heteroskedasticity. In large samples, the distribution of $\underline{u}_i$ is approximately normal with mean zero and variance $\frac{V_i}{T}$. We can compute standard errors and t-statistics for each of the sample moments and we can compute a χ^2 test statistic for each security,

$$\chi^2_{(k)} = T \underline{u}_i V_i^{-1} \underline{u}_i ,$$

where k is the degrees of freedom and is equal to the number of instruments in $\underline{z}_{i,t-1}$. One can also construct a χ^2 test for the time-series moments associated with a subset of securities, but the covariance matrix is not invertible if the number of sample moments exceeds T.

In terms of existing empirical models in the literature, this estimation is most closely related to the signal extraction problem which arises in statistics and econometrics. The standard problem is a linear model of the form $y_t = x_t + e_t$, where x_t, the variable of interest, is observed with error. A common example is extracting expected inflation rates from observed inflation rates with a model of how expectations are formed. Our problem is reversed because we observe interest rates, the conditional expectation of the MRS. The model from which we extract estimates of the MRS is nonlinear, but we have restrictions on a large number of moments and the resulting estimator is linear. The MRS estimator employs an asset pricing relation and does not require a complete description of how expectations on key

variables are formed. We have implicitly used the assumption that expectations are formed rationally. This model has similarities and dissimilarities with the factor model in the arbitrage pricing theory (APT). This model relies on a large cross-sectional sample of security returns as does the model of Connor and Korajczyk (1986), but instead of trying to identify a number of underlying factors in a linear return-generating model, we are trying to extract estimates of the MRS which arises in intertemporal CAPM's. The MRS variable may be driven by a number of factors or state variables in the economy, and it is conceivable that there might be a resulting linear factor model for stock returns.

From the model in equation (1), we can derive the following risk-return relationship:

$$E_{t-1}(R_{i,t}) - R_{Ft} = - \mathrm{Cov}_{t-1}[\eta_t, R_{i,t}], \qquad (6)$$

which states that the risk premium on a security is negatively related to the conditional covariance of the security return and the MRS. With estimates of this covariance, one can perform cross-sectional regression tests similar to those which have been used to test the standard CAPM and the APT as in Fama and MacBeth (1973) and Roll and Ross (1980). Estimating this conditional covariance could be difficult, and we suspect that the risk premia and conditional covariances change over time. Equation (6) does imply that average risk premia for securities should equal the negative of the unconditional covariance, but we have used this relationship in forming our estimate of $\underline{\eta}$. Our procedure of testing the model on a subset of security

returns incorporates this relationship when we include a constant in the set of instrumental variables.

II. Empirical Results with the MRS Model

The first step in the empirical analysis is to estimate the MRS with the estimator developed in Section I. The data for the estimation stage include monthly returns for the period 1926-85. We use the returns on one-month Treasury bills, long-term Treasury bonds, and long-term corporate bonds computed by Ibbotson and Sinquefield plus the returns on stocks taken from the CRSP tapes. We use three instruments for each security return. The three instruments for T-bill and T-bond returns are a constant, $(1+R_{Ft})$ and $(1+R_{B,t-1})/(1+R_{Ft})$, where R_{Ft} is the T-bill return and R_{Bt} is the T-bond return. The three instruments for corporate bonds are a constant, $(1+R_{Ft})$, and $(1+R_{c,t-1})/1+R_{Ft})$, where R_{ct} is the corporate bond return. The three instruments for each stock return, R_{it}, are a constant, $(1+R_{Ft})$, and $(1+R_{i,t-1})/(1+R_{Ft})$. The risk free return has been included because numerous studies have found small, but significant correlations between stock returns and risk-free interest rates (sometimes labeled expected inflation).

If we were to use the entire period 1926-85, we would need to invert a very large matrix in order to compute $\hat{\underline{\eta}}$. To make the estimator feasible, we have split the sample into two periods: February 1926 to December 1955 and January 1956 to December 1985. The samples include all common stocks with complete return series on the monthly CRSP tape. For the 1926-55 period, we have 251 companies, and for the 1956-85 period, we have 384 companies. Securities with missing return

observations can be included in the estimator of Section I, but the programming would be more difficult and we have omitted those securities. With the Treasury securities, the corporate bond series, and the common stocks, we have 762 time-series moments for the 1926-55 period and 1161 time-series moments for the 1956-85 period. The number of cross-sectional moments is equal to T for each period so that the total number of sample moments is 1121 for the 1926-55 period and 1521 for the 1956-85 period. The estimates of the MRS range from a low of .555 to a high of 1.826, and the estimates are plotted in Figure 1A and 1B.

The second step of the empirical analysis is to use the estimated MRS series to test the intertemporal CAPM relation. We test the restriction $E\left\{ \left[\left(\frac{\lambda_t}{\lambda_{t-1}}\right)(1+R_{it}) - 1\right] \underline{z}_{i,t-1} \right\} = 0$ by testing whether the time-series sample moments are close to zero.[3] The results of these tests are contained in Tables I-III. The first tests are performed on the NYSE-CRSP value weighted return index, Treasury bonds, corporate bonds, and the η_t series. In Table I, we present the results for the entire period 1927-85, and in Table II we present results for a more recent period 1952-85, which is frequently used in empirical studies. For the NYSE index, we include the following four instruments: a constant, $(1+R_{m,t-1})/(1+R_{Ft})$, $(1+R_{Ft})$, and $\frac{\overline{D}_{m,t-1}}{P_{m,t-1}}$ where $\overline{D}_{m,t-1}$ is the accumulation of cash dividends over the months $(t-12)$ through $(t-1)$.[4] We have included the dividend yield because several studies have documented correlations between stock returns and lagged dividend yields.[5] All of the t statistics in Table I are small indicating that none of the sample moments are significantly different from zero. The magnitudes of all of the sample moments are small:

the largest absolute value is .0006. Two of the sample moments are effectively set to zero by the estimation of $\underline{\eta}$. None of the χ^2 statistics are significant, and we conclude that these security returns satisfy the restrictions of the intertemporal CAPM.

In Table II, the results for the period 1952-85 are mixed. The sample moments are small and none of the t statistics are significant, but the χ^2 statistics are all significant at the 1% level. Each χ^2 test statistic is a test of the null hypothesis that all of the sample moments for the security are zero. Either the χ^2 statistics are reflecting something that we do not see in the t statistics, or the χ^2 statistics may be the result of problems in the inversion of the variance matrices for the sample period.

In Table III, we present a summary of the results of tests on 100 security returns for the period 1952-85. We have selected the first 100 securities on the monthly CRSP tape that have complete return histories from 1951 to 1985. The securities are listed in Table VII at the end of the paper. We use four instruments for each security including the dividend yield.[6] The results are similar to those in Table II. None of the t statistics are significant for the first three instruments: the constant, $(1+R_{i,t-1})/(1+R_{Ft})$, and $(1+R_{Ft})$. Three of the t statistics for the dividend yield are significant at the 5% level and two of these are significant at the 1% level. Thirty-six of the one-hundred χ^2 statistics are significant at the 5% level and fourteen of these are signficant at the 1% level. The results for individual stocks generally support the intertemporal CAPM restrictions, but there is some ambiguity with respect to the χ^2 statistic.

In Table IV, we present the tests of the intertemporal CAPM restrictions in an alternative framework by using more conventional regression tests. The regressions have been computed with the NYSE index, and we estimate three different equations. An alternative method of testing the sample moments is to regress $\frac{\lambda_t}{\lambda_{t-1}}(1+R_{mt})$ on a constant and the instrumental variables:

$$\frac{\lambda_t}{\lambda_{t-1}}(1+R_{mt}) = \beta_0 + \beta_1 \left(\frac{1+R_{m,t-1}}{1+R_{Ft}}\right) + \beta_2 (1+R_{Ft}) + \beta_3 \frac{\overline{D}_{m,t-1}}{P_{m,t-1}} + e_t$$

Under the null hypothesis of the intertemporal CAPM, β_0 should equal one and β_1, β_2, and β_3 should equal zero. A regression of $\frac{\lambda_t}{\lambda_{t-1}}(R_{mt}-R_{Ft})$ on a constant and lagged variables known at time t-1 should produce a zero intercept and zero coefficients on all lagged variables. In the third regression, we regress $(R_{mt}-R_{Ft})$ on a constant and lagged variables known as t-1. This third regression is not a test of the intertemoral CAPM, but it is a test of a conventional model that is used in empirical studies. By placing restrictions on the distributions of the MRS and security returns, Hansen and Singleton (1983) and others have derived a result that expected excess returns should be constant and excess returns should be unpredictable. The intercept in the third regression measures the risk premium and the coefficients on the lagged variables should be zero. In panel A of Table IV we have the regression results for the period 1927-85. The R^2's for all three equations are small and all of the tests indicate acceptance of the intertemporal CAPM restrictions and the stronger restrictions implied by the excess return model. In Panel B, we present the same three regressions for the more recent period 1952-85. The t statistics for

some of the coefficients on individual variables are significant and the joint χ^2 test statistics are significant at very low marginal significance levels. The regression tests indicate rejection of both the intertemporal CAPM restrictions and the restrictions of the excess return model for the more recent peirod. It is interesting to note that over the longer period the model restrictions are accepted, but are rejected during the more recent sub-period. At this point, we conjecture that there may be something unique during the period 1952-85 which is averaged out or disappears over a longer time period. Given the results for the longer period, we conclude that the data generally support the restrictions of the intertemporal CAPM.

In addition to the formal statistical tests, we have made some calculations to compare our estimates of the MRS with proxies that arise from the log utility model and the consumption-based models and we have run tests for conditional heteroskedasticity on some of our key variables. With a specific utility function, one can develop alternative measures of the MRS. For example, the log utility model implies $(\frac{\lambda_t}{\lambda_{t-1}}) = \frac{W_{t-1}}{W_t}$, where W_t is end of period nominal wealth, and if we have a market portfolio which represents aggregate wealth, then $(\frac{\lambda_t}{\lambda_{t-1}}) = \frac{1}{1+R_{mt}}$. In the consumption-based CAPM, we are using the intertemporal envelope relation that marginal utility of real wealth equals marginal utility of real consumption. With the power utility of consumption function (constant relative risk aversion), we have the following model for the MRS:

$$\frac{\lambda_t}{\lambda_{t-1}} = \rho(\frac{c_t}{c_{t-1}})^{-\alpha} \frac{I_{t-1}}{I_t},$$

where c_t is real per capital consumption, I_t is the consumption price index, ρ is the time preference parameter, and α is the coefficient of relative risk aversion.

We calculate three proxies for the MRS. The first one is based on $\frac{1}{1+R_{mt}}$ and uses the return on the NYSE-CRSP value weighted portfolio as a proxy for the return on the market portfolio. The second and third proxies are based on the consumption-based model and we use consumption expenditures for nondurables plus services divided by population to measure per capita consumption. The implicit price deflator for consumption of non-durables plus services is used as the consumption price index. Because the estimates in the literature for α range from near one to four, we have used two proxies based on $\alpha=1$ and $\alpha=4$.[7] The comparisons of our estimates of the MRS with these three proxies are made with quarterly data: hence we use quarterly returns on the NYSE portfolio and quarterly consumption data.[8] The matrix of correlation coefficients for these four series is presented in Table V, and Figures 2-4 contain scatter diagrams for each of the three proxies versus our estimate of the MRS. The table and the three figures indicate that there is a high correlation between $\frac{1}{1+R_{mt}}$ and our estimate of the MRS series, but almost no relationship between the consumption-based proxies and the estimated MRS series. It should be no surprise that the MRS series is highly correlated with the inverse of one plus the rate of return on the NYSE portfolio; the MRS estimate is based on security returns that are included in the NYSE portfolio. But note that the correlation is not perfect and that the MRS series varies more than the market return proxy.

We have also calculated the correlation coefficient between $(\frac{\hat{\lambda}_t}{\lambda_{t-1}})$ and $(1+R_{mt})^{-1}$ using monthly data, and the correlation coefficient is .986, very close to the number for the quarterly data. The low correlation between the consumption-based proxies and our estimated MRS indicate that these versions of the consumption-based CAPM perform quite poorly. There are several problems with the consumption data and the time-additive separable utility function that may account for this poor performance. These results, as well as others in the literature, suggest either that the consumption-based model does not adequately characterize security returns or that the consumption data are not appropriate for testing the consumption-based models.

In Section I, we noted that the estimates and empirical tests allow for the possibility of conditional heterocedasticity in the data. In Table VI we present some evidence of autoregressive conditional heteroscedasticity in some of our key variables. We apply Engle's (1982) LaGrange multiplier test: in this application, we regress the square of the error term on three lagged values of itself and TR^2 is approximately distributed as a χ^2 with three degrees of freedom. We have used several simple models for η_t, the market excess return, and R_{Ft}. The two models for η_t are

$$\eta_t = 1 + u_t$$

$$\ln \eta_t = \beta_0 + u_t.$$

We also consider two simple models for the market excess return:

$$R_m - R_{Ft} = \beta_0 + u_t$$

$$\ln \frac{(1+R_{mt})}{(1+R_{Ft})} = \beta_0 + u_t.$$

For this risk-free interest rate, we estimate a 6'th order autoregressive model. We do find evidence of conditional heteroscedasticity in the error terms for all five models. Even though we have tested for only one form of autoregressive conditional heteroskedasticity, the results indicate that there is some conditonal heteroskedasticity in all three variables. These results indicate one possible explanation for the rejection of asset pricing relations in models which require additional assumptions on the distributions of security returns and the MRS.[9]

III. Summary and Conclusions

In the first part of this paper, we develop a method for using security returns to extract estimates of the MRS, which we treat as an unobservable. The estimator makes use of the unlimited number of sample moments available on security returns to indentify and estimate the underlying MRS series which should be common across all securities. The estimates are then used to test the relationship which arises in the intertemporal CAPM by applying the test to a subset of securities. We find that the results generally support the intertemporal CAPM. In addition we make some comparisons between our estimates of the MRS and estimates or proxies that arise in models that require specific assumptions on investor preferences. The empirical observations suggest that the consumption-based CAPM's do not adequately characterize asset returns. The poor performance of the consumption-based

models may be due to either poor measurements of the necessary consumption series or to highly restrictive assumptions on the form of the utility function. Our results, particularly those in Table V and Figures 2-4, suggest that tests and applications of intertemporal CAPM's should emphasize financial market data such as interest rates and security returns instead of consumption data.

APPENDIX A

From the budget constraints of intertemporal consumption-investment decisions, we have the following set of necessary conditions for an economy with N agents who are neither identical nor share the same information sets:

$$J_w^k(t)p_t - E[J_w^k(t+1)\,(p_{t+1}+d_{t+1}) \mid \phi_t^k] = 0,\ k=1,\ldots,N,$$

where prices and dividends are in real terms and we have suppressed the index on p_t and d_t for different securities. For examples, see Lucas (1978) and Breeden (1979). Restating the model in nominal terms, we have

$$\lambda_t^k P_t - E[\lambda_{t+1}^k(P_{t+1}+D_{t+1}) \mid \phi_t^k] = 0,\ k=1,\ldots,N \qquad \text{(A-1)}$$

where λ_t^k is the marginal utility of real wealth times the consumption price deflator for individual k. These pricing equations are aggregated across all N investors and equilibrium prices are formed so that investors as a group are willing to hold all the shares outstanding. To avoid boundary conditions for some investors, we must assume unrestricted short selling. Given equilibrium market prices, the relationships in (A-1) should be satisfied. Private information plays a role in the price formation, but we do not investigate that issue here. Instead we consider the role of market information defined as follows:

$$\phi_t^m = \phi_t^1 \cap \phi_t^2 \cap \cdots\cdots \cap \phi_t^N,$$

that is, market information includes information that is known by all agents. Agents may or may not know the marginal utility of wealth parameters, λ_t, for other agents. If they have this information, the

model is simplified. We consider the case in which agents do not, but instead we form an expectation about these preference parameters conditional on market information. Our next step is to take the expectation of each equation in (A-1) conditional on ϕ^m_t, noting that $\phi^m_t \subset \phi^k_t$ for k=1,...,N.

$$P_t E(\lambda^k_t \mid \phi^m_t) - E[\lambda^k_{t+1}(P_{t+1}+D_{t+1}) \mid \phi^m_t] = 0$$

Then we aggregate across the N investors.

$$P_t \sum_{k=1}^{N} E(\lambda^k_t \mid \phi^m_t) - \sum_{k=1}^{N} E[\lambda^k_{t+1}(P_{t+1}+D_{t+1}) \mid \phi^m_t] = 0$$

For the second term, we have

$$E[\sum_{k=1}^{N} \lambda^k_{t+1}(P_{t+1}+D_{t+1}) \mid \phi^m_t] = E[(P_{t+1}+D_{t+1})(\sum_{k=1}^{N} \lambda^k_{t+1}) \mid \phi^m_t]$$

Let $\lambda_t \equiv \sum_{k=1}^{N} E(\lambda^k_t \mid \phi^m_t)$. By the law of interated expectations,

$$E[(P_{t+1}+D_{t+1})(\sum_{k=1}^{N} \lambda^k_{t+1}) \mid \phi^m_t] = E\,\{E[(P_{t+1}+D_{t+1})(\sum_{k=1}^{N} \lambda^k_{t+1}) \mid \phi^m_{t+1}] \mid \phi^m_t\},$$

and it follows that

$$\lambda_t P_t - E[\lambda_{t+1}(P_{t+1}+D_{t+1}) \mid \phi^m_t] = 0, \qquad \text{(A-2)}$$

where $\lambda_{t+1} \equiv \sum_{k=1}^{N} E(\lambda^k_{t+1} \mid \phi^m_{t+1})$. If agents know current values of λ^k_t for all investors, then this result follows with $\lambda_t \equiv \sum_{k=1}^{N} \lambda^k_t$. (A-2)

implies the relationship studied in the paper with the interpretation that E_t is the expectation conditional on market information, ϕ_t^m, and λ_t is an aggregation of the marginal utility of wealth variable across agents. It is not necessary to assume that agents are identical and share all information to derive this result.

APPENDIX B

In this appendix, we establish the consistency of $\hat{\underline{\eta}}$ as an estimator of $\underline{\eta}$.

$$\hat{\underline{\eta}} = A^{-1}\underline{b}$$

$$\hat{\underline{\eta}} - \underline{\eta} = A^{-1}\underline{b} - \underline{\eta} = A^{-1}(\underline{b} - A\underline{\eta})$$

$A\underline{\eta} - \underline{b}$ is simply the set of first-order conditions for the minimization problem with the true η's. We examine the probability limit as K and T get large together. Alternatively we can get convergence in probability by fixing T and letting $K \to \infty$, but here we show convergence as $K,T \to \infty$. Recall the elements of the TxT matrix A. The off-diagonal terms (t,s) for t,s=1,...,T are

$$\frac{1}{T^2} \sum_{i=1}^{K} \left(\frac{1+R_{it}}{1+R_{Ft}}\right)\left(\frac{1+R_{is}}{1+R_{Fs}}\right)\underline{z}'_{i,t-1}\underline{z}_{i,s-1}.$$

The diagonal terms (t=1,...,T) are

$$\left(\frac{1}{K} \sum_{i=1}^{K} \frac{1+R_{it}}{1+R_{Ft}}\right)^2 + \frac{1}{T^2} \sum_{i=1}^{K} \left(\frac{1+R_{it}}{1+R_{Ft}}\right)^2 \underline{z}'_{i,t-1}\underline{z}_{i,t-1}.$$

We assume that the cross-sectional moments settle down to cross-sectional averages. The following cross-sectional moments converge in probability to constants:

$$\frac{1}{K} \sum_{i=1}^{K} \frac{1+R_{it}}{1+R_{Ft}} \quad \text{and}$$

$$\frac{1}{K} \sum_{i=1}^{K} \left(\frac{1+R_{it}}{1+R_{Ft}}\right)\left(\frac{1+R_{is}}{1+R_{Fs}}\right)\underline{z}'_{i,t-1}\underline{z}_{i,s-1}$$

for t,s=1,...,T. As T and K get large, A converges to a diagonal matrix. It follows that A^{-1} also converges to a diagonal matrix whose elements are the inverses of $(\frac{1}{K}\sum_{i=1}^{K}\frac{1+R_{it}}{1+R_{Ft}})^2$. Each element of $A^{-1}(\underline{b}-A\underline{\eta})$, in the limit as K,T get large, is the product of a constant and

$$- e_t(\frac{1}{K}\sum_{i=1}^{K}\frac{1+R_{it}}{1+R_{Ft}}) - \frac{1}{T}\sum_{i=1}^{K}(\frac{1+R_{it}}{1+R_{Ft}})\underline{u}_i'\underline{z}_{i,t-1} \tag{B-1}$$

$\operatorname{plim}_{K\to\infty} e_t = 0$ and $\operatorname{plim}_{T\to\infty} \underline{u}_i = \underline{0}$ by standard results on sample moments. The first term in (B-1) converges in probability to zero. The second term involves a sum and each term in the sum converges in probability to zero. Because we are letting K and T go to infinity at the same rate, we are effectively taking the average of terms with zero probability limits. Hence, the second term in (B-1) also converges in probability to zero. It follows that $\operatorname{plim}_{K,T\to\infty} A^{-1}(\underline{b}-A\underline{\eta}) = \underline{0}$ and we conclude that $\operatorname{plim}_{K,T\to\infty} \hat{\underline{\eta}} - \underline{\eta} = \underline{0}$, which establishes the consistency of $\hat{\underline{\eta}} = A^{-1}\underline{b}$ as an estimator of $\underline{\eta}$.

Footnotes

[1]Connor and Korajczyk (1986) have developed an empirical approach to the APT that also uses the large cross sections of security returns.

[2]This alternative estimator requires a two-step procedure. First, we use the cross-sectional moments to compute initial estimates of $\underline{\eta}$. The initial estimates are used to calculate the simple variance estimates. In the second step, we re-estimate $\underline{\eta}$ using the variances and all of the sample moments.

[3]The cross-sectional moments are not used in these tests because of the difficulties involved in computing variances for these sample moments.

[4]We use the NYSE returns calculated with dividends.

[5]For results cited on divided yields, see Keim (1985) and Summers (1985).

[6]The stock returns and dividend yields have been adjusted for stock splits and stock dividends. $\overline{D}_{i,t-1}$ is calculated in the same manner as $\overline{D}_{m,t-1}$.

[7]For each value of α, we have estimated a corresponding ρ following the method used in Grossman and Shiller. We find that we must use data from 1947 to 1983 in order to get a ρ estimate less than one for $\alpha = 4$. For $\alpha = 1$, we use $\rho = .9851$, and for $\alpha = 4$, we use $\rho = .9982$.

[8]The quarterly estimate of our $(\frac{\lambda_t}{\lambda_{t-1}})$ is simply computed as the product of the three corresponding monthly estimates.

[9]For an example, see the tests on differences between two security returns in Hanesen and Singleton (1983).

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FIGURE 1 A

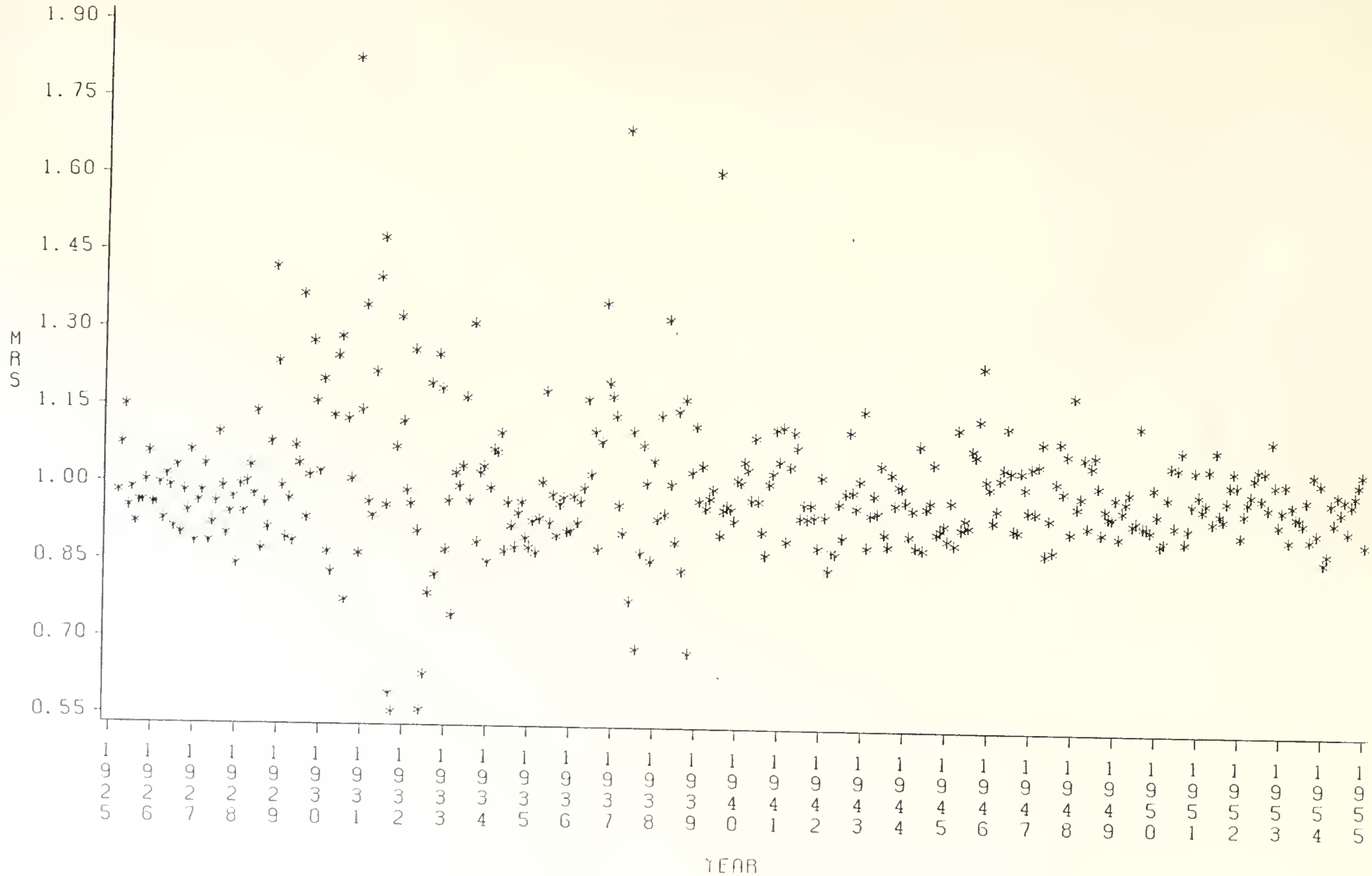
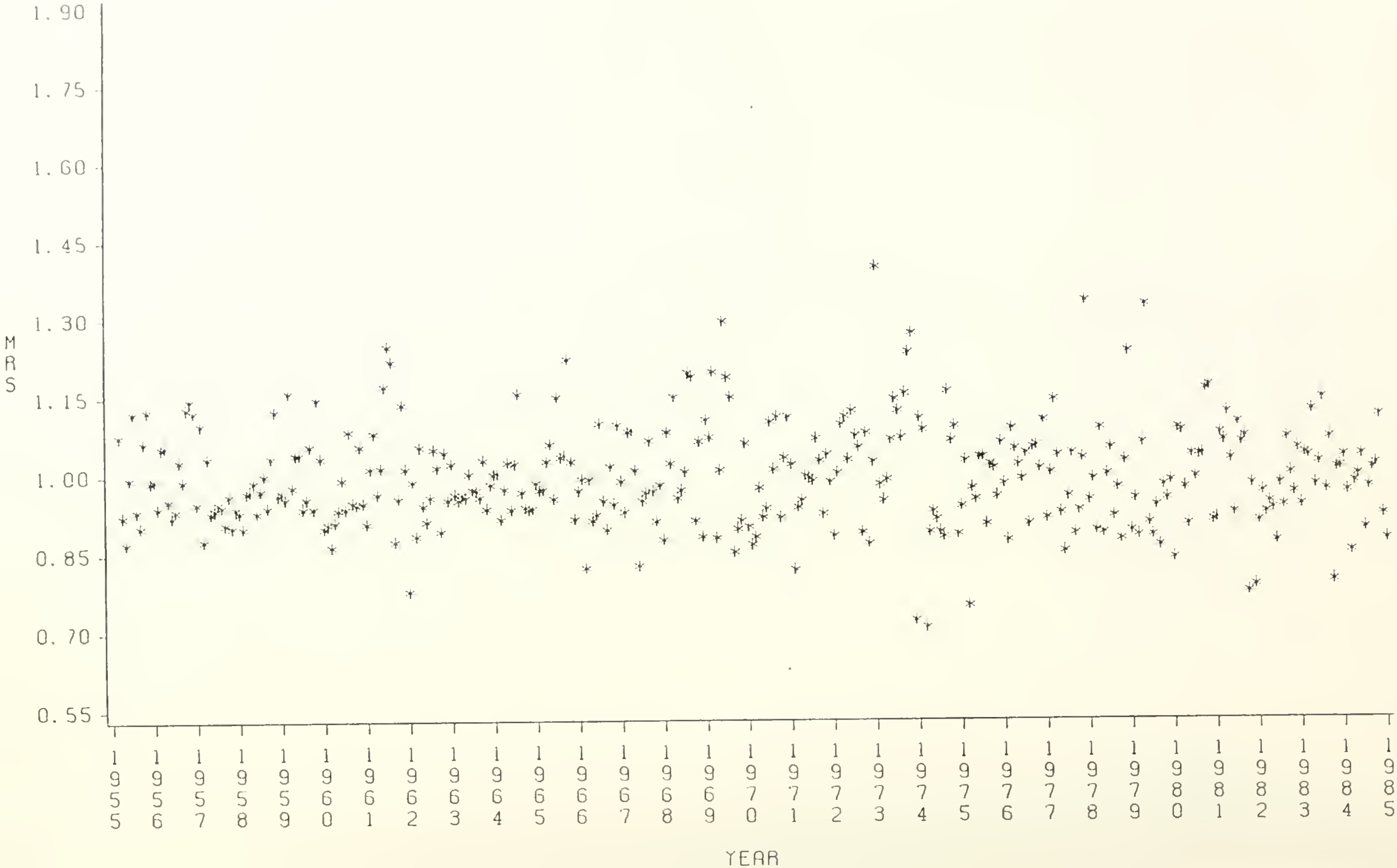

MRS
1.90
1.75
1.60
1.45
1.30
1.15
1.00
0.85
0.70
0.55
1925 1926 1927 1928 1929 1930 1931 1932 1933 1934 1935 1936 1937 1938 1939 1940 1941 1942 1943 1944 1945 1946 1947 1948 1949 1950 1951 1952 1953 1954 1955
YEAR

FIGURE 1 B



1.90
1.75
1.60
1.45
1.30
1.15
1.00
0.85
0.70
0.55
MRS
1955 1956 1957 1958 1959 1960 1961 1962 1963 1964 1965 1966 1967 1968 1969 1970 1971 1972 1973 1974 1975 1976 1977 1978 1979 1980 1981 1982 1983 1984 1985
YEAR

Table I

Tests on the Intertemporal CAPM

$$E(\underline{u}_i) = E\left\{\frac{1}{T}\sum_{t=1}^{T}\left[\frac{\lambda_t}{\lambda_{t-1}}(1+R_{it}) - 1\right]\underline{z}_{i,t-1}\right\} = 0$$

Sample Period: January 1927 to December 1985, T = 708

A. NYSE-CRSP Value Weighted Return Index

Instrument	Sample Moment	Standard Error	t Statistic
Constant	-9.231×10^{-5}	.002244	-.04
$\frac{(1+R_{m,t-1})}{(1+R_{Ft})}$	-4.875×10^{-4}	.002266	-.22
$1+R_{Ft}$	-8.161×10^{-5}	.002250	-.04
$\frac{\overline{D}_{m,t-1}}{P_{m,t-1}}$	-6.010×10^{-5}	.0001225	-.49

$\chi^2(4) = 5.67$

B. Long Term Treasury Bonds

Instrument	Sample Moment	Standard Error	t Statistic
Constant	1.708×10^{-4}	.004339	.04
$\frac{(1+R_{B,t-1})}{(1+R_{Ft})}$	-3.184×10^{-5}	.004351	-.01
$1+R_{Ft}$	1.873×10^{-4}	.004348	.04

$\chi^2(3) = 5.33$

Table I (continued)

C. Long Term Corporate Bonds

Instrument	Sample Moment	Standard Error	t Statistic
Constant	5.836×10^{-4}	.004317	.14
$\frac{(1+R_{c,t-1})}{(1+R_{Ft})}$	4.069×10^{-4}	.004316	.09
$1+R_{Ft}$	6.000×10^{-4}	.004326	.14

$\chi^2(3) = 4.63$

D. $$E(\underline{u}_i) = E\left\{\frac{1}{T}\sum_{t=1}^{T}[\eta_t - 1]\,\underline{z}_{i,t-1}\right\} = 0$$

Instrument	Sample Moment	Standard Error	t Statistic
Constant	-2.854×10^{-5}	.004433	-.01
$\frac{(1+R_{B,t-1})}{(1+R_{Ft})}$	-2.447×10^{-4}	.004443	-.06
$1+R_{Ft}$	-9.283×10^{-6}	.004447	.00

$\chi^2(3) = 5.36$

Table II

Tests of the Intertemporal CAPM

Sample Period: February 1952 to December 1985, T = 407

A. NYSE-CRSP Value Weighted Return Index

Instrument	Sample Moment	Standard Error	t Statistic
Constant	-.0004146	.002657	-.16
$\frac{(1+R_{m,t-1})}{(1+R_{Ft})}$	-.0006512	.002630	-.25
$1+R_{Ft}$	-.0003980	.002671	-.15
$\frac{\overline{D}_{m,t-1}}{P_{m,t-1}}$	$-5.966\text{x}10^{-5}$	.0001098	-.54

$\chi^2(4) = 25.92$

B. Long Term Treasury Bonds

Instrument	Sample Moment	Standard Error	t Statistic
Constant	-003045	.004477	-.68
$\frac{(1+R_{B,t-1})}{(1+R_{Ft})}$	-.003477	.004464	-.79
$1+R_{Ft}$	-.003021	.004500	-.67

$\chi^2(3) = 18.78$

Table II (Continued)

Tests of the Intertemporal CAPM

Sample Period: February 1952 to December 1985, T = 407

C. Long Term Corporate Bonds

Instrument	Sample Moment	Standard Error	t Statistic
Constant	-.002789	.004398	-.63
$\frac{(1+R_{c,t-1})}{(1+R_{Ft})}$	-.003139	.004380	-.72
$1+R_{Ft}$	-.002765	.004420	-.63

$\chi^2(3) = 13.77$

D. $E(\underline{u}_i) = E\left\{\frac{1}{T} \sum_{t=1}^{T} [\eta_t - 1]\, \underline{z}_{i,t-1}\right\} = 0$

Instrument	Sample Moment	Standard Error	t Statistic
Constant	-.001970	.004654	-.42
$\frac{(1+R_{B,t-1})}{(1+R_{Ft})}$	-.002433	.004639	-.52
$1+R_{Ft}$	-.001940	.004678	-.41

$\chi^2(3) = 16.89$

Table III

Tests of the Intertemporal CAPM, 100 Common Stocks

Sample Period: February 1952 to December 1985, T = 407

Instrument	Range for t Statistic	Number of times $\lvert t \rvert > 1.96$	Number of times $\lvert t \rvert > 2.576$
Constant	-1.78 to 1.74	0	0
$\frac{(1+R_{i,t-1})}{(1+R_{Ft})}$	-1.90 to 1.84	0	0
$1+R_{Ft}$	-1.78 to 1.74	0	0
$\frac{\bar{D}_{i,t-1}}{P_{i,t-1}}$	-2.61 to 2.12	3	2

$\chi^2(4)$ Statistic

Range: .76 to 21.39

Number of Times $\chi^2(4) > 9.49$: 36

Number of Times $\chi^2(4) > 13.28$: 14

Table IV

Regression Tests

A. Sample Period: January 1927 to December 1985, T = 708

$$(1)\ \frac{\lambda_t}{\lambda_{t-1}}(1+R_{mt}) = \underset{(.8993)}{.2878} - \underset{(.0617)}{.1283}\frac{(1+R_{m,t-1})}{(1+R_{Ft})} + \underset{(.8877)}{.8539}(1+R_{Ft}) - \underset{(.2397)}{.3361}\frac{\overline{D}_{m,t-1}}{P_{m,t-1}} + e_t$$

$R^2 = .02$

D.W. = 2.01

Test of $\beta_1 = \beta_2 = \beta_3 = 0$, $\chi^2(3) = 7.05$

Test of $\beta_0 = 1$ and $\beta_1 = \beta_2 = \beta_3 = 0$, $\chi^2(4) = 7.07$

$$(2)\ \frac{\lambda_t}{\lambda_{t-1}}(R_{mt} - R_{Ft}) = \underset{(.01173)}{-.0001222} + \underset{(.05914)}{.09028}\frac{\lambda_{t-1}}{\lambda_{t-2}}(R_{m,t-1} - R_{F,t-1}) - \underset{(.7971)}{.9225}R_{Ft} + \underset{(.2702)}{.05923}\frac{\overline{D}_{m,t-1}}{P_{m,t-1}} + e_t$$

$R^2 = .01$

D.W. = 2.00

Test of $\beta_1 = \beta_2 = \beta_3 = 0$, $\chi^2(3) = 4.14$

Test of $\beta_0 = \beta_1 = \beta_2 = \beta_3 = 0$, $\chi^2(4) = 4.15$

$$(3)\ R_{mt} - R_{Ft} = \underset{(.01362)}{-.009310} + \underset{(.07077)}{.1242}(R_{m,t-1} - R_{F,t-1}) - \underset{(.7382)}{.9927}R_{Ft} + \underset{(.3083)}{.3956}\frac{\overline{D}_{m,t-1}}{P_{m,t-1}} + e_t$$

$R^2 = .03$

D.W. = 2.00

Test of $\beta_1 = \beta_2 = \beta_3 = 0$, $\chi^2(3) = 6.76$

Table IV (continued)

B. Sample Period: February 1952 to December 1985, T = 407

(1) $$\frac{\lambda_t}{\lambda_{t-1}}(1+R_{mt}) = \underset{(1.0205)}{-2.1772} - \underset{(.0711)}{.1185}\frac{(1+R_{m,t-1})}{(1+R_{Ft})} + \underset{(1.0100)}{3.3180}(1+R_{Ft}) - \underset{(.3059)}{.9285}\frac{\overline{D}_{m,t-1}}{P_{m,t-1}} + e_t$$

R^2 = .045

D.W. = 1.93

Test of $\beta_1 = \beta_2 = \beta_3 = 0$, $\chi^2(3) = 21.76$*

Test of $\beta_0 = 1$ and $\beta_1 = \beta_2 = \beta_3 = 0$, $\chi^2(4) = 24.90$*

(2) $$\frac{\lambda_t}{\lambda_{t-1}}(R_{mt} - R_{Ft}) = \underset{(.00881)}{-.01644} + \underset{(.05997)}{.07275}\frac{\lambda_{t-1}}{\lambda_{t-2}}(R_{m,t-1} - R_{F,t-1}) - \underset{(.8398)}{.34896}R_{Ft} + \underset{(.2305)}{.8391}\frac{\overline{D}_{m,t-1}}{P_{m,t-1}} + e_t$$

R^2 = .065

D.W. = 1.97

Test of $\beta_1 = \beta_2 = \beta_3 = 0$, $\chi^2(3) = 28.48$*

Test of $\beta_0 = \beta_1 = \beta_2 = \beta_3 = 0$, $\chi^2(4) = 32.58$*

(3) $$R_{mt} - R_{Ft} = \underset{(.008849)}{-.01597} + \underset{(.05644)}{.04138}(R_{m,t-1} - R_{F,t-1}) - \underset{(.7944)}{3.2189}R_{Ft} + \underset{(.2407)}{.8900}\frac{\overline{D}_{m,t-1}}{P_{m,t-1}} + e_t$$

R^2 = .058

D.W. = 1.97

Test of $\beta_1 = \beta_2 = \beta_3 = 0$, $\chi^2(3) = 26.81$*

NOTE: Standard errors are in parentheses. We have allowed for conditional heteroskedasticity in computing the standard errors and χ^2 statistics.

* Significant at the 5% level.

** Significant at the 1% level.

Table V

Correlation Matrix
Quarterly Data, 1952:II to 1985:IV

Variables:

(1) $\frac{\lambda_t}{\lambda_{t-1}}$, MRS

(2) $\frac{1}{1+R_{mt}}$

(3) Consumption-Based Proxy for MRS, Relative Risk Aversion = 1

(4) Consumption-Based Proxy for MRS, Relative Risk Aversion = 4

Variable	(1)	(2)	(3)	(4)
(1)	1.000			
(2)	.985	1.000		
(3)	- .096	- .083	1.000	
(4)	.094	.090	.729	1.000

FIGURE 2

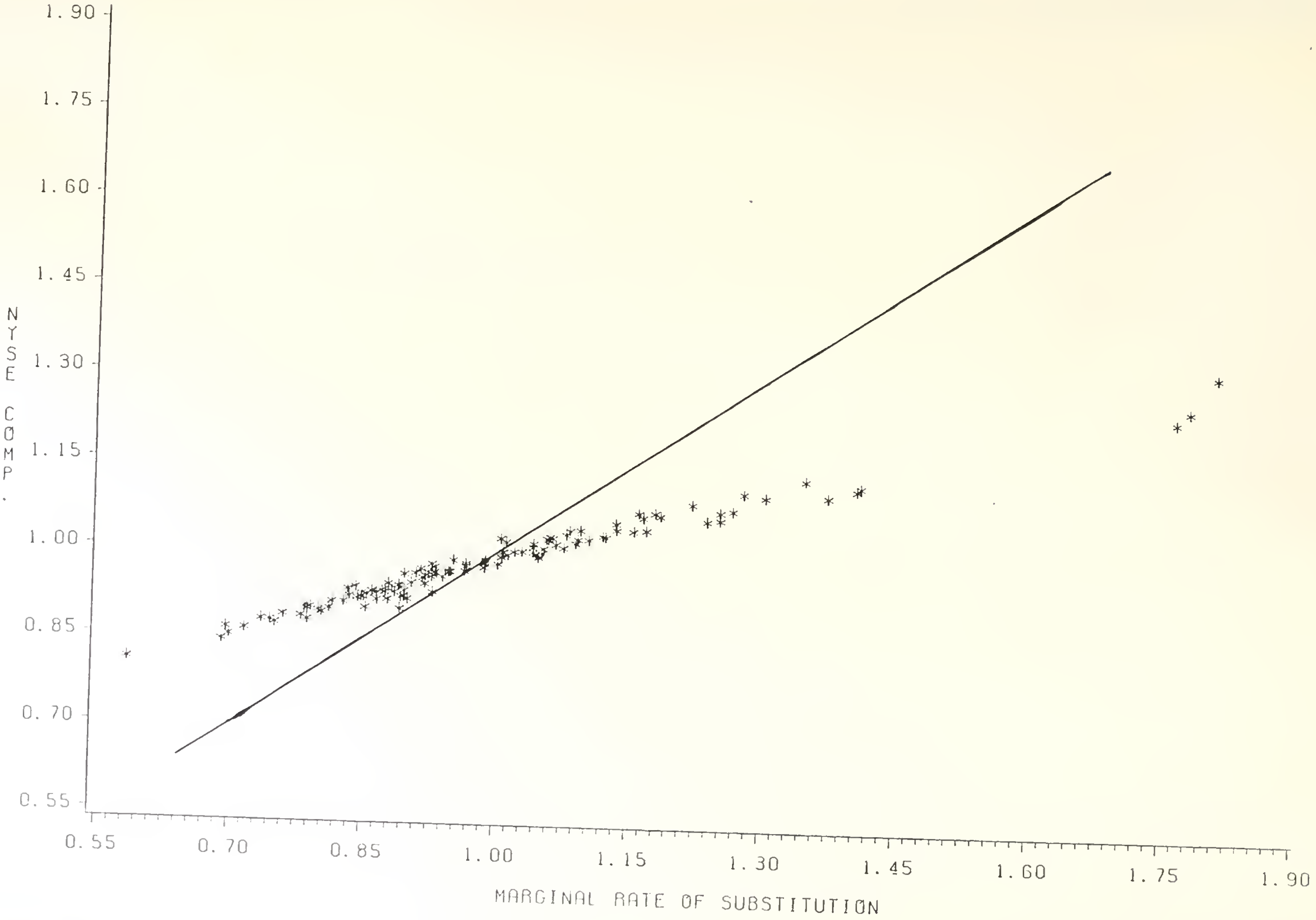
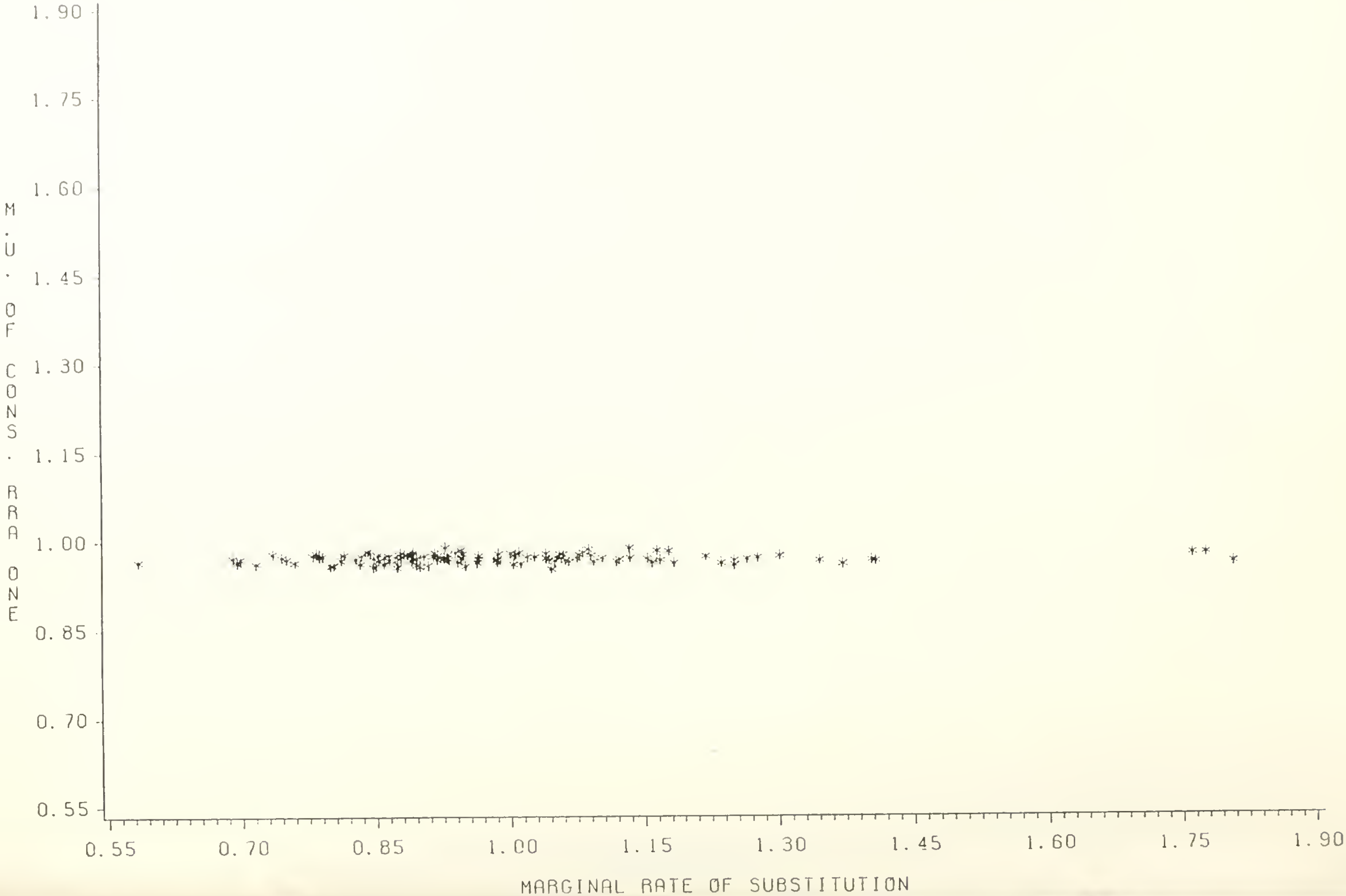

1.90
1.75
1.60
1.45
1.30
1.15
1.00
0.85
0.70
0.55
NYSE COMP.
0.55
0.70
0.85
1.00
1.15
1.30
1.45
1.60
1.75
1.90
MARGINAL RATE OF SUBSTITUTION

FIGURE 3



M.U. OF CONS. RRA ONE
1.90
1.75
1.60
1.45
1.30
1.15
1.00
0.85
0.70
0.55
0.55
0.70
0.85
1.00
1.15
1.30
1.45
1.60
1.75
1.90
MARGINAL RATE OF SUBSTITUTION

FIGURE 4

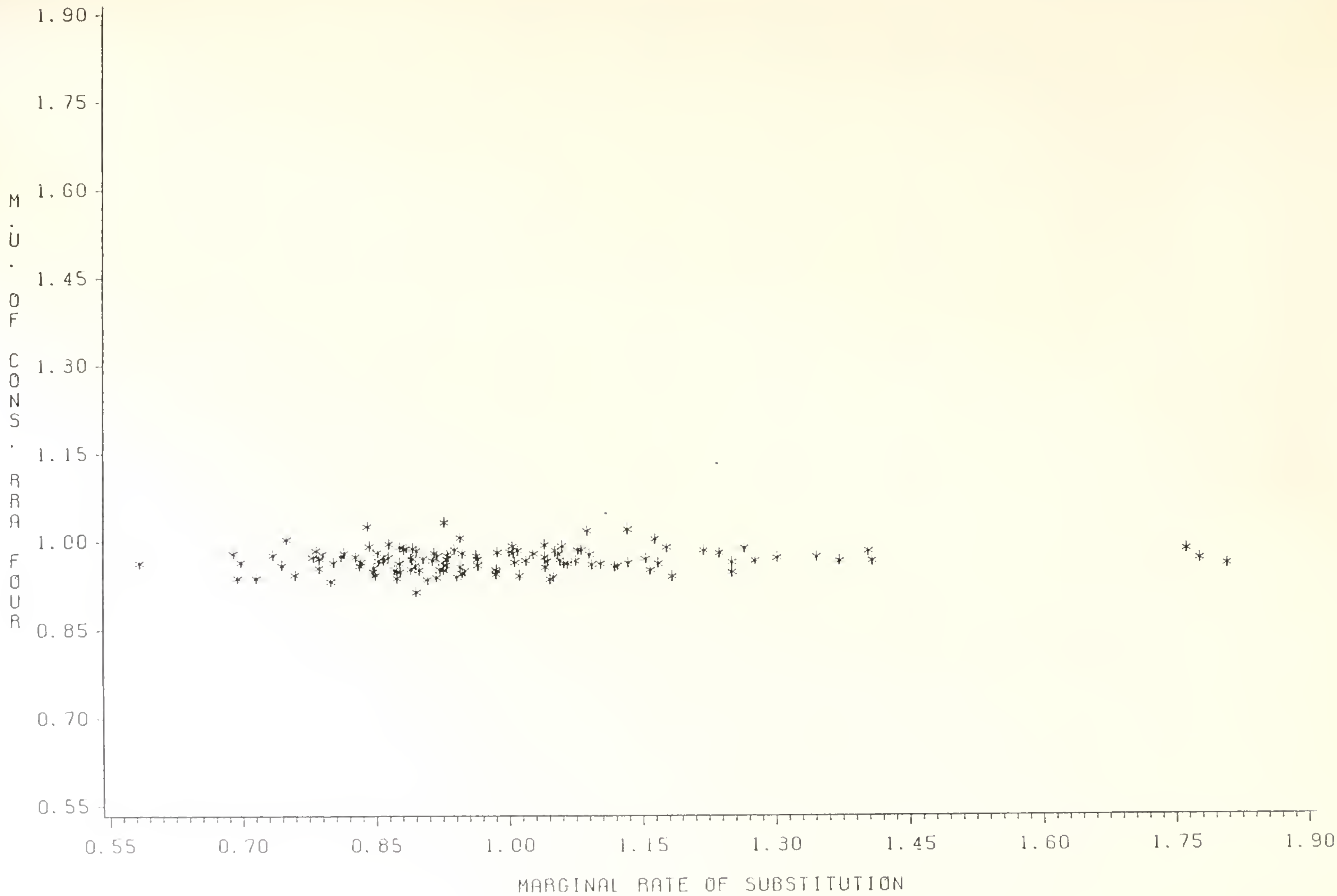

M.U. OF CONS. RRA FOUR
1.90
1.75
1.60
1.45
1.30
1.15
1.00
0.85
0.70
0.55
0.55 0.70 0.85 1.00 1.15 1.30 1.45 1.60 1.75 1.90
MARGINAL RATE OF SUBSTITUTION

Table VI

Tests for Autoregressive Conditional Heteroskedasticity

All Models of the Form: $u_t^2 = a_0 + a_1 u_{t-1}^2 + a_2 u_{t-2}^2 + a_3 u_{t-3}^2$

	a_0	a_1	a_2	a_3	R^2	TR^2
$\eta_t - 1. = u_t$						
	.006572 (.001049)	.06841 (.04978)	.08661 (.04972)	.09252 (.04978)	.024	9.86*
$\ln \eta_t = -.010583 + u_t$						
	.005948 (.001001)	.06379 (.04946)	.09624 (.04934)	.01460 (.04947)	.040	16.32**
$R_{mt} - R_{F,t-1} = .005290 + u_t$						
	.001076 (.000184)	.09984 (.0494)	.09893 (.0495)	.1472 (.0495)	.052	20.89**
$\ln \left(\frac{1+R_{mt}}{1+R_{F,t-1}}\right) = .004459 + u_t$						
	.001084 (.000181)	.1148 (.04961)	.0954 (.04973)	.1222 (.0496)	.047	19.15**
R_{Ft}, 6'th Autoregression						
	2.792×10^{-7} (8.054×10^{-8})	.2240 (.0504)	.2346 (.0502)	−.0239 (.0504)	.129	51.42**

NOTE: Standard errors are in parentheses.
*indicates significance at the 5% level.
**indicates significance at the 1% level.

$$R_{Ft} = .0001258 + .8489\, R_{F,t-1} - .0652\, R_{F,t-2} + .2004\, R_{F,t-3} - .1656\, R_{F,t-4}$$
$$+ .1077\, R_{F,t-5} + .0486\, R_{F,t-6} + u_t$$

Table VII

Stocks Used in Tests of the Intertemporal CAPM in Table III

		CUSIP Number
1.	AMR	001765
2.	Abbot Labs	002824
3.	Acme Cleveland	004626
4.	Adams Express	006212
5.	Adams Millis	006284
6.	Alcan Aluminum	013716
7.	Alleghany	0171761
8.	Allegheny Power Systems	017411
9.	Allegheny International	017372
10.	Allied Stores	019519
11.	Allis Chalmers	019645
12.	Amax	023127
13.	American Bakeries	024069
14.	American Brands	024703
15.	American Broadcasting Company	024735
16.	American Cyanamid	025321
17.	American Electric Power	025537
18.	American Home Products	026609
19.	American Motors	027627
20.	American Standards	029717
21.	AT&T	030177
22.	American Water Works	030411
23.	Ametek	031105
24.	Amoco	031905
25.	Ampco-Pittsburgh	032037
26.	Amsted Industries	032177
27.	Anchor Hocking	033047
28.	Anderson Clayton	033609
29.	Archor Daniels Midland	039483
30.	Armco	042170
31.	Armstrong World Industries	042476
32.	Arvin Industries	043339
33.	Asarco	043413
34.	Ashland Oil	044540
35.	Associated Dry Goods	045573
36.	Atlantic City Electric	048303
37.	Atlantic Richfield	048825
38.	Atlas	049267
39.	Baltimore Gas & Electric	059165
40.	Becor Western	075873
41.	Belding Heminway	077491
42.	Bell & Howell	077851
43.	Beneficial	081721

44. Benguet	081851
45. Bethlehem Steel	087509
46. Black & Decker	091797
47. Boeing	097023
48. Borden	099599
49. Borg Warner	099725
50. Briggs & Stratton	109043
51. Bristol Myers	110097
52. Brooklyn Union GAs	114259
53. Brown Group	115657
54. Brunswick	117043
55. Burlington Industries	121691
56. Burroughs	122781
57. CBS	124845
58. CPC International	126149
59. Callahan Mining	131069
60. Canadian Pacific	136440
61. Carolina Power & Light	144141
62. Carpenter Tractor	144285
63. Caterpillar Tractor	149123
64. Celanese	150843
65. Central & South West	152357
66. Central Hudson Gas & Electric	153609
67. Certainteen	156879
68. Champion International	158525
69. Chevron	166751
70. Chicago Pneumatic Tool	167898
71. Chris Craft	170520
72. Chyrsler	171196
73. Cilcorp	171794
74. Cincinnati Gas & Electric	172070
75. Cincinnati Milacron	172172
76. Clark Equipment	181396
77. Cleveland Electric	186108
78. Cluett Peabody	189486
79. Coca Cola	191216
80. Colgate Palmolive	194162
81. Collins & Aikman	194828
82. Columbia Gas Systems	197648
83. Combustion Engineering	200273
84. Commonwealth Edison	202795
85. Consolidated Edison NY	209111
86. Consolidated Natural Gas	209615
87. Consumers Power	210615
88. Corning Glass	219327
89. Crane	224399
90. Crown Cork & Seal	228255
91. Crown Zellerbach	228669
92. Culbro	229890
93. Curtiss Wright	231561

Table VII (continued)

94.	Cyclops	232525
95.	Dayco	239577
96.	Dayton Power & Light	240019
97.	Deere	244199
98.	Delmarva Power & Light	247109
99.	De Soto	250595
100.	Detroit Edison	250847